Exercises 1 Calculations

1 Use your calculator to evaluate the following to the number of decimal places indicated.

a) $125.67 + (47.31 - 72.64)$ to 2 dp.

b) $125.67 - (47.31 - 72.64)$ to 2 dp.

c) $9.77 + 3.256 \times 7.418$ to 3 dp.

d) $4.512 \times (3.654 + 6.71)$ to 3 dp.

e) $\dfrac{(17.51 + 11.42)}{6.89}$ to 2 dp.

f) $9.51 + 4.67^3$ to 2 dp.

g) $\sqrt{\dfrac{25.431}{8.012} + 2.151}$ to 3 dp.

h) $\dfrac{1}{6.95} + \sqrt[3]{21.145}$ to 3 dp.

i) $5.61 - 2.35 \times (7.22 + 3.45)$ to 2 dp.

j) $7.823^{\frac{2}{3}}$ to 3 dp.

2 If $x = 2.34 \times 10^4$ and $y = 1.76 \times 10^{-3}$ evaluate each of the following giving the answer to 3 significant figures.

i) x^2 ii) $x^2 y$ iii) $\dfrac{x}{y}$ iv) $\dfrac{100y}{x}$ v) $\sqrt{xy}$

3 In experiments on plant morphology the famous biologist Mendel hypothesised that the ratio of round seed to angular seed should be 3:1. If the hypothesis is true and 436 seeds are considered, how many will be expected to be round and how many will be expected to be angular.

4 In order to dilute a bottle of orange squash, it is recommended to mix squash and water in the ratio 2:7. How much squash should be put into a glass that holds 270mL.

5 A newly qualified science graduate is given a job with a starting salary of £13500. After one year in the job the graduate receives a 6.5% salary rise. What is the new salary?

6 You have 42mL of a 2.5M solution of NaCl, but need a 0.7M solution. How many mL of 0.7M NaCl can you make?

7 You need 150mL of 0.9M NaOH solution and you have a 3M stock solution. How would you make up the solution?

8 Urea lysis buffer contains the following ingredients:

9.9g urea (RMM 60.06)/ 100mL
22g SDS (RMM 288.4)/100mL
77mL 5M NaCl stock solution/100mL
2.5mL 0.2M EDTA stock solution/100mL
15mL 1M Tris-HCl pH8.0 stock solution/100mL.

What are the final molar concentrations of *each* of the components of this buffer?

9 If there is 1 mole of a substance dissolved in 0.4 m^3 of a solution, (i) obtain the concentration of the solution in mol/m^3, (ii) what amount (i.e. how many moles) will there be in each dm^3 of this solution (of the same concentration)?

Consider the chemical equation

$$H_2SO_4 + Na_2CO_3 = Na_2SO_4 + CO_2 + H_2O.$$

10 If a sample of 40.0 cm^3 of H_2SO_4 solution (concentration 0.077 mol/dm^3) is just neutralised by 5.2 cm^3 of Na_2CO_3 solution, calculate the (molar) concentration of the Na_2CO_3.

11. You are provided with a stock solution of the monosodium salt of α ketoglutaric acid (chemical formula $C_5H_5O_5Na$) of concentration 4gL^{-1}.

You are required to make up a set of standards in 1mL volumetric flasks, using the volumes given in the table below.

Calculate the concentration of the salt in each standard both in gL^{-1} and mmol/L.

Volume of the salt (4gL^{-1}) (mL)	0.00	0.04	0.08	0.20	0.40	0.80	1.00

12. Repeat question 11 using glucose (chemical formula $C_6H_{12}O_6$).

Exercise 1 Answers

1	a)	100.34	b)	151.00	c)	33.923	d)	46.762
	e)	4.20	f)	111.36	g)	±2.308	h)	2.909
	i)	-19.46	j)	3.941				
2	i)	5.48×10^{8}	ii)	9.64×10^{5}	iii)	1.33×10^{7}		
	iv)	7.52×10^{-6}	v)	±6.42				

Exercise 1 Answers continued

3 Round 327, angular 109.

4 60mL.

5 £14377.50

6. 150mL

7. 45mL of stock solution made up to 150mL

8. Urea 1.65M, SDS 0.76M, NaCl 3.85M, EDTA 0.005M, Tris 0.15M.

9. (i) 2.5 mol/m^3 (ii) 0.0025 mol.

10 0.592 mol dm^{-3}.

11.

Substance conc. (g/L)	Substance conc. (mmol/L)
0.00	0.00
0.16	0.95
0.32	1.90
0.80	4.76
1.60	9.52
3.20	19.04
4.00	23.80

12.

Substance conc. (g/L)	Substance conc. (mmol/L)
0.00	0.00
0.16	0.89
0.32	1.78
0.80	4.44
1.60	8.88
3.20	17.77
4.00	22.21

Exercises 2 Transposition of Formulae

Transpose the following for the variable indicated

1. $x = 2y - 5$ for y.

2. $F = \frac{9}{5}C + 32$ for C.

3. $i = \frac{nE}{R + nr}$ for E.

4. $T = 2\pi\sqrt{\frac{l}{g}}$ for l.

5. $x = 2 + \frac{1}{t}$ for t.

6. $t = \frac{WL}{A + 2L}$ for L.

7. $\alpha = \frac{l - l_o}{l_o \theta}$ for l_0.

8. $\eta = \frac{1 + \alpha t}{1 - \alpha t}$ for t.

9. $x = \sqrt{\frac{y}{1 - y}}$

Exercise 2 Answers

1. $y = \frac{x + 5}{2}$

2. $C = \frac{5}{9}(F - 32)$

3. $E = \frac{i(R + nr)}{n}$

4. $l = \left(\frac{T}{2\pi}\right)^2 g$ or $l = \frac{T^2}{4\pi^2} g$

5. $t = \frac{1}{x - 2}$

6. $L = \frac{At}{W - 2t}$

7. $l_o = \frac{l}{\alpha\theta + 1}$

8. $t = \frac{\eta - 1}{\alpha(\eta + 1)}$

9. $y = \frac{x^2}{x^2 + 1}$

Exercises 3 Logarithms

1 Using the definition of logarithms write down the value of the following

(i) $\log_2 16$ (ii) $\log_3 243$ (iii) $\log_5 3125$ (iv) $\log_8 128$

2 Using the definition of logarithms find x when:

(i) $\log_{10} x = 3$ (ii) $\log_{10} x = -1$ (iii) $\log_2 x = \frac{1}{2}$ (iv) $\log_3 x = 2$

3 Which of the following is **true** and which is **false**?

(i) $\log 2 + \log 3 = \log 6$

(ii) $ab = \log a + \log b$

(iii) $\log 250 - \log 125 = \log 125$

(iv) $2\log 4 = \log 16$

(v) $\log 5 = -\log\left(\frac{1}{5}\right)$

4 Find x if $\log x = \log 12 + 3\log 2$ (ans $x = 96$)

5 Find x if $2\ln x - 1 = 5$ (ans $x = 20.086$)

6 Find x if $\log_{10} x = \log_{10} y - 3$ (ans $x = \frac{y}{1000}$)

7 Find x if $\log(x+3) - \log(x-5) = \log 2$ (ans $x = 13$)

8 Find x if $12.5 = 2.5^x$ (ans $x = 2.756$)

9 Find x if $4^{x-1} = 9.1$ (ans $x = 2.593$)

10 Find x if $5^x = 7^{1-x}$ (ans $x = 0.547$)

Exercises 3 Logarithms continued

11 In a chemical solution let $[H_3O^+]$ denote the concentration of hydrogen ions measured in moles/L (moles per litre). The pH of the solution is defined as:

$$pH = -log_{10}[H_3O^+]$$

Find the pH of solutions in which

(i) $[H_3O^+] = 3.1 \times 10^{-4}$

(ii) $[H_3O^+] = 0.21 \times 10^{-9}$ (ans (i) 3.51 (ii) 9.68)

Use the spreadsheet Excel as well as your calculator to do the following questions.

12 Calculate the *pH* for the following hydrogen ion concentrations.
Give your answer to 2 decimal places.

1. 2.0×10^{-2}
2. 4.8×10^{-4}
3. 1.0×10^{-7}
4. 2.0×10^{-10}
5. 6.3×10^{-14}
6. 0.50
7. 1.00
8. 1.50
9. 3.02

13 Calculate the hydrogen ion concentrations of the following pH solutions.

1. 2.00
2. 2.20
3. 2.40
4. 2.60
5. 3.00
6. 3.40
7. 6.60
8. 7.30
9. 9.70
10. 11.50
11. 12.00
12. 13.00
13. 2.10
14. 2.11
15. 2.12
16. 2.13

From your last 4 calculations, can you see that two decimal places in the pH does not really give the concentration precise to 3 significant figures?

Use the spreadsheet Excel as well as your calculator to do these questions.

Exercises 4 Functions

1 Rearrange the following so that they are in the form which expresses y explicitly as a function of x.

(i) $x+4y=5$ (ii) $3-4y-5x^2=0$ (iii) $xy+4x-7y=7$

(iv) $y+x^2y-x^2=0$

2 If $f(x)=x^3+7x^2-1$ evaluate $f(-1),\ f(2),\ f(0).$

3 In a test for blood sugar metabolism, conducted over a time interval, the amount of sugar in the blood (mg/100m*l*) was a function of time t (measured in hours) and given by:

$$A(t)=3.9+0.2t-0.1t^2$$

Find the amount of sugar in the blood,

(a) at the beginning of the test
(b) 1 hour after the beginning
(c) 2.5 hours after the beginning.

4 If $y=30e^{-0.2x}$ where e is the base of natural logarithms, tabulate the function for $x = 0, 1, 2, 3, 4, 5$ and hence draw a graph of the function over this range of values of x. Use Excel for this question

Exercises 4 Answers

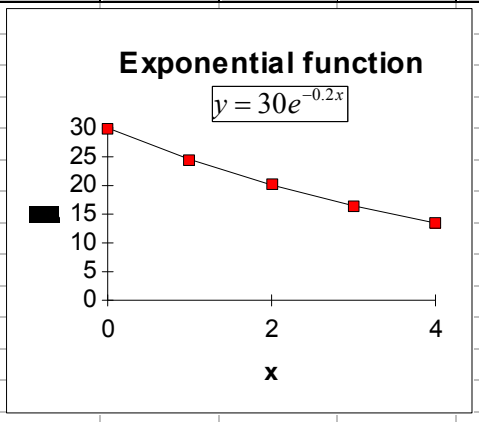
1 (i) $y=\dfrac{5-x}{4}$ (ii) $y=\dfrac{3-5x^2}{4}$ (iii) $y=\dfrac{7-4x}{x-7}$ (iv) $y=\dfrac{x^2}{1+x^2}$

2 $f(-1)=5,\quad f(2)=35,\quad f(0)=-1.$

3 (a) $A(0)=3.9,$ (b) $A(1)=4$ (c) $A(2.5)=3.775.$

4

	A	B	C	D	E	F	G
1	x	y					
2	0	30					
3	1	24.56192					
4	2	20.1096					
5	3	16.46435					
6	4	13.47987					
7							
8							
9							
10							
11							
12							
13							
14							



Exercises 5 Straight Line

1 Do the points (1,-1) and (3,12) lie on the line $y = 6x - 7$?

2 Find the equation of the straight line passing through the points (3,2) and (7,8).

3 Find the y-coordinate of the points whose x-coordinates are 2 and -3 which lie on the line $y = -x + 2$.

4 Find the slope and intercept of the following straight lines

(i) $6y - 2x + 3 = 0$ (ii) $5x - 2y - 8 = 0$

In question 5 you are expected to use Excel

5 The amount of ascorbic acid in a given volume of solution may be estimated by measuring the extent to which it decolourises the blue starch iodine complex (using a spectrophotometer). A number of solutions were made up containing known amounts of ascorbic acid and the meter reading for each found.

Meter reading (x)	5.9	4.8	3.7	2.4	0.9	0.0
Amount of acid (y)	150	300	450	600	750	900

Show that x and y are connected by the equation $y = mx + c$ and determine m and c

.A sample of milk was tested for ascorbic acid and the meter reading was 1.3. Use your equation to estimate the ascorbic acid content of the milk.

Exercises 5 Answers

1 (1,-1) YES, (3,12) NO ((3,11) lies on the line).

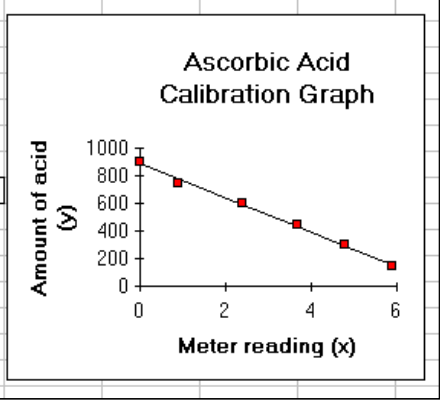
2 $y = 1.5x - 2.5$

3 When $x = 2, y = 0$ and when $x = -3, y = 5$

4 (i) slope $m = \frac{1}{3}$, intercept $c = -\frac{1}{2}$ (ii) slope $m = \frac{5}{2}$, intercept $c = -4$

5

	A	B	C	D	E
1	x	y	y calc	error	error^2
2	5.9	150	161.9	11.875	141.013
3	4.8	300	297.3	-2.7225	7.41218
4	3.7	450	432.7	-17.32	299.98
5	2.4	600	592.7	-7.2987	53.2711
6	0.9	750	777.3	27.341	747.54
7	0	900	888.1	-11.875	141.013
8				Total =	1390.23
9	m =	-123			
10	c =	888.1			
11					
12	When x = 1.3		y =	728.1	
13					
14					
15					



Exercises 6 Quadratics and Exponentials

1 Factorise the following quadratic equations and hence solve them

(i) $2x^2 - x - 3 = 0$ (ii) $x^2 + 10x + 21 = 0$

(iii) $3x^2 + 5x - 2 = 0$ (iv) $4x^2 - 4x + 1 = 0$

2 Use the quadratic formula to solve the following

(i) $x^2 - 5x + 2 = 0$ (ans 4.561, 0.438)

(ii) $3x^2 + 7x + 3 = 0$ (ans -1.768, -0.566)

3 A population of microorganisms is doubling in size every 75 minutes. If it initially weighs 0.1gm, what will its weight be after 5 hours and after 500 minutes?. How long does it take to reach a weight of 0.5gm? Assume a growth law of the form

$$n = n_0 e^{kt}$$

where n is the mass at time t, n_0 is the mass at $t=0$ and k is a constant.

(ans 1.6gm, 10.16gm, 174.1mins)

4 A piece of charcoal is found to have C^{14} content equal to 77% of the natural level found in living matter. How old is the charcoal?

(ans 2101 years)

5 The half-life of radium is 1590 years. If 10gm of radium are left for 1000 years, how much will remain?

(ans 6.47gm)

6 Under ideal conditions in a synthetic medium, each cell in a culture of **E.coli** divides into two cells every 50 minutes. If the growth is according to the law

$$n = n_0 e^{kt}$$

where n_0 is the biomass at $t=0$, n is the biomass at time t minutes and k is a constant, calculate the time taken for the biomass to increase

(i) by a factor of 8

(ii) by a factor of 32

(iii) by a factor of 5.

(ans (i)150 mins (ii)250mins (iii)116.1mins)

Exercises 6 continued

7 A drug is administered intravenously to a subject. The blood carries the drug to the proper cells, and any excess drug is removed by the liver. Experiments show that the elimination of the drug from the circulatory system may be described mathematically by the exponential function

$$c = c_0 e^{-kt}$$

where c_0 denotes the concentration, in moles per litre, of the drug immediately after the injection and c is the concentration of the drug in the blood, moles per litre, at time t after the injection and k is a constant.

If 5% of the drug has been removed from the system in 1 hour, calculate the time for the concentration of the drug to be half its initial value.

(ans 13.5 hours)

Exercises 7 Reduction of Laws to Linear Form

1 Show how the following can be transformed to straight line form $Y = mX + c$. In each case state what X, Y, m and c are in terms of $x, y, a,$ and b.

(i) $y = a + b\sqrt{x}$ (ii) $y = ax + bx^2$

(iii) $y = \frac{a}{x} + bx$ (iv) $y = a + \frac{b}{x^2}$

In questions 2, 3 and 4 you are expected to use Excel. Try to fit the laws by linearizing them and by fitting them directly. Compare the answers.

2 Values obtained for the solubility s mol m^{-3} of a certain chemical in water at different temperatures $T\ ^oC$ are given below

$T\ ^oC$	20	25	30	35	40	45	50
s mol m^{-3}	32.8	38.7	45.7	54.1	64.0	75.7	89.5

Show by plotting a suitable graph that the relationship between s and T is of the form

$$s = 16 + aT + bT^2$$

Obtain values for the constants a and b.

3 During a study of oxygen concentrations a record was kept of the oxygen uptake, U, during the oxydisation of succinate for a period of time t (minutes)

t (mins)	100	150	200	250	300	400	500	600
U	96	59	42	32.5	26	18.6	14.3	11.5

It was thought that these quantities obeyed a law of the form

$$U = at^n$$

Verify this graphically and determine possible values for the constants a and n.

4 In a subject whose fasting blood glucose level is 100mg/100ml, 50g of glucose were injected intravenously. At 20 minute intervals blood samples were taken and the blood glucose levels measured. The following results were recorded

minutes after injection (t)	20	40	60	80	100	120
blood glucose (y)	180	149	131	119	112	108

Test the suggested law

$$y = ae^{bt} + 100$$

and determine approximate values for a and b.

Exercises 7 Answers

1 (i) $y = a + b\sqrt{x}$ (ii) $\frac{y}{x} = a + b\,x$

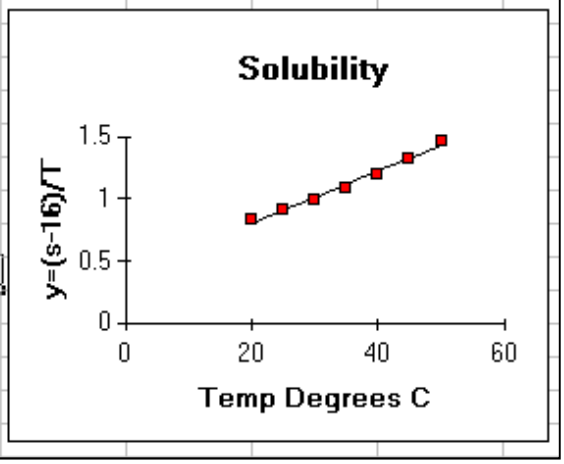
ie $y = a + bX$ ie $Y = a + bX$

(iii) $xy = a + bx^2$ (iv) $y = a + b.\frac{1}{x^2}$

ie $Y = a + bX$ ie $y = a + bX$

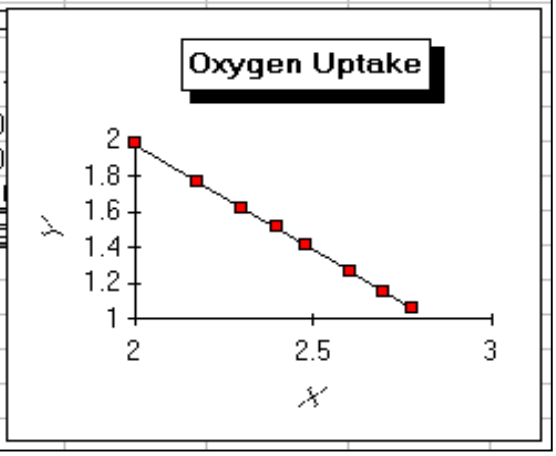
2

	A	B	C	D	E	F
1	T	s	y=(s-16)/T	y calc	error	error^2
2	20	32.8	0.84	0.803	0.03711	0.00138
3	25	38.7	0.908	0.908	0.00021	4.2E-08
4	30	45.7	0.99	1.013	-0.0227	0.00052
5	35	54.1	1.088571	1.118	-0.029	0.00084
6	40	64	1.2	1.223	-0.0225	0.00051
7	45	75.7	1.326667	1.327	-0.0007	5.6E-07
8	50	89.5	1.47	1.432	0.03768	0.00142
9					Total =	0.00466
10			a =	0.383		
11			b =	0.021		
12						
13						
14						



3

	A	B	C	D	E	F	G
1	t	U	X=logt	Y=logU	Y calc	error	error^2
2	100	96	2	1.98227	1.9805	0.001736	3.01E-06
3	150	59	2.1761	1.77085	1.7724	-0.00156	2.44E-06
4	200	42	2.301	1.62325	1.6247	-0.00	
5	250	32.5	2.3979	1.51188	1.5102	0.001	
6	300	26	2.4771	1.41497	1.4166	-0.00	
7	400	18.6	2.6021	1.26951	1.269	0.000	
8	500	14.3	2.699	1.15534	1.1544	0.000	
9	600	11.5	2.7782	1.0607	1.0608	-0.00	
10						Tota	
11			n =	-1.1819			
12			log a =	4.34432			
13			a =	22096.4			
14							
15							
16							



Exercise 7 Answers continued

4

	A	B	C	D	E	F	G	H	I
1	t	y	y-100	Y=ln(y-100)	Y calc	error	error^2		
2	20	180	80	4.3820	4.3616	0.0205	0.00042		
3	40	149	49	3.8918	3.8				
4	60	131	31	3.4340	3.4				
5	80	119	19	2.9444	2.9				
6	100	112	12	2.4849	2.5				
7	120	108	8	2.0794	2.0				
8									
9			b =	-0.023176					
10			$c = \ln a$	4.8250917					
11			$\therefore a =$	124.598					
12									
13									
14									
15